

$\vec{p} = m\vec{v}$ $\vec{F}_L = m(\vec{v} - \vec{u})$ $\vec{F}t$ (Ns/kg.m/s) $\Sigma \vec{F} = m\vec{a}$
 $\vec{F} = \frac{m(\vec{v} - \vec{u})}{t}$ \vec{v}_{av} (m/s) \vec{F} (N) $\vec{F} \rightarrow \square$
 $\Sigma \vec{F} = \frac{\Delta(m\vec{v})}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t}$ \vec{a}_{av} (m/s²) $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$
 $W = mg$ $\Sigma \vec{F} = \frac{\Delta(m\vec{v})}{\Delta t}$ $\vec{v}_{av} = \frac{\Delta \vec{s}}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$

Newton's Formula

$F = W$ $f_{s,max} = \mu_s N$ $f_k = \mu_k N$
 $\Sigma \vec{F} = m\vec{a}$ $\frac{F}{M} = g$
 $\Sigma \vec{F} = T - mg$ $F_c = \frac{mv^2}{r} = m\omega^2 r = ma_c$
 $\vec{a} \uparrow$ $\vec{s} = \vec{v}t$ ($\vec{a} = 0$) ω (Rad/s)
 $\vec{v} = \vec{u} + \vec{a}t$ $t_{n \rightarrow c} = \frac{V_n}{g} = \frac{V_n}{10}$ $\vec{v}_x = \vec{v} \cos \theta$
 $T = m(g + a)$ $\Sigma \vec{F} = 0 \rightarrow \vec{a} = 0 \rightarrow \vec{v}$ $\vec{v}_y = \vec{v} \sin \theta$

FORCE AND NEWTON'S LAWS

Learning Outcomes

Understand the vector definition of force.

Use trigonometric identities to resolve forces into components.

Define net force, external force, and system.

Define normal, tension, weight and friction forces.

Understand Newton's first law of motion.

Understand Newton's second law of motion.

Apply Newton's second law to determine acceleration

Understand Newton's third law of motion.

Distinguish between the internal and external forces for a system

Apply Newton's third law to solve problems of motion.

Apply Newton's laws of motion to solve problems involving a variety of forces.

Discuss the general characteristics of friction.

Describe the various types of friction.

Calculate the magnitude of static and kinetic friction.

State Hooke's law and explain using a graphical representation.

Discuss the three types of deformations such as changes in length, sideways shear and changes in volume.

Describe with examples the young's modulus, shear modulus and bulk modulus.

Concepts

F = Force in magnitude

\mathbf{F} = Force as a vector

a = Acceleration in magnitude

\mathbf{a} = Acceleration as a vector

m = mass

μ_k = Coefficient of kinetic friction

μ_s = Coefficient of static friction

TYPE OF FORCES

W = Weight

F_n = Normal force

T = Tension

f_k = Kinetic friction

f_s = Static friction

$g = 9.8 \text{ m/s}^2$

a_x = acceleration in x

a_y = acceleration in y

F_x = x component of the Force

F_y = y- component of the Force

\mathbf{F}_{net} = Net force as vector

F_{net} = Magnitude of \mathbf{F}_{net}

Σ = Sum

F_{12} = Force on object 1 due to object 2

Units

SI UNITS

acceleration is in meters per second square " m/s^2 "

Force in Newton's "N"

mass is in kg

Angle in degrees or radians

Formulas and Constants

$$W = mg$$

$$F_{\text{net}} = ma$$

$$\mathbf{F}_{\text{net}} = m\mathbf{a}$$

$$\sum F_x = F_{\text{Net}}(x)$$

$$\sum F_y = F_{\text{Net}}(y)$$

$$F^2 = F_x^2 + F_y^2$$

$$\tan\theta = F_y/F_x$$

$$\theta = \text{atan}(F_y/F_x)$$

$$F_{\text{net}}^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta$$

cosine theorem for two forces, θ is the angle between F_1 and F_2

Conditions for an object in Equilibrium $\sum F_x = 0$ and $\sum F_y = 0$

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \text{ Action-Reaction}$$

$$F_{\text{external}} = m_{\text{total}} a$$

$$F = -k \Delta x \text{ Hooke's Law}$$

$$f_k = \mu_k F_n$$

$$f_s(\text{max}) = \mu_s F_n$$

$$0 \leq f_s \leq f_s(\text{max})$$

$F_D = \frac{1}{2} C_D \rho A v^2$ Drag force, C_D is a constant depending of shape, A is the area.

VECTOR REPRESENTATIONS OF FORCES

Graphical

Magnitude/Angle

Components

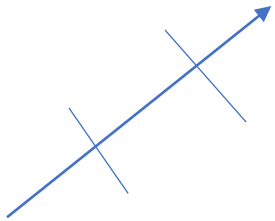
Vector Representation

3 units at 45 degrees

$F_x = 3 \cos 45 = 2.1$ units

$F = 2.1 \mathbf{i} + 2.1 \mathbf{j}$

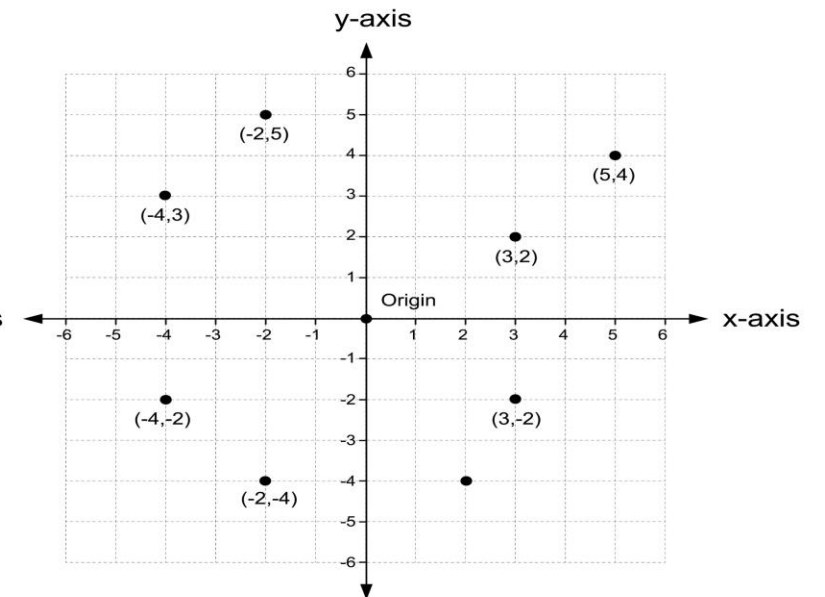
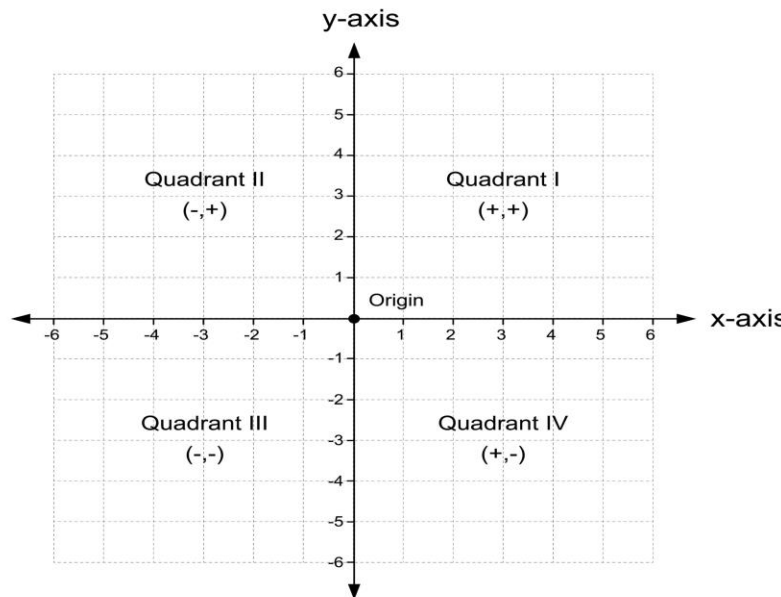
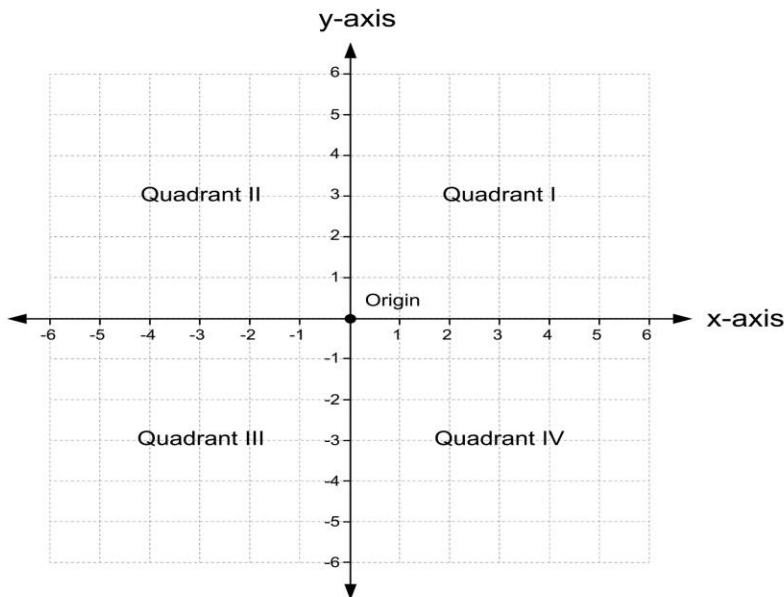
$F_y = 3 \sin 45 = 2.1$ units



QUADRANTS AND DIRECTIONS

First quadrant F_x is +, F_y is +, second quadrant F_x is -, F_y is -

third quadrant F_x and F_y are -, fourth quadrant F_x is + F_y is -



Cartesian coordinate system

CLASS WORK VECTOR REPRESENTATION OF FORCE

$F_x = F \cos(\theta)$ only when the angle is measured with respect to x-axis

$F_y = F \sin(\theta)$ only when the angle is measured with respect to x-axis

$F = |\mathbf{F}| = \sqrt{F_x^2 + F_y^2}$ Pythagorean Theorem

$\tan\theta = F_y/F_x$

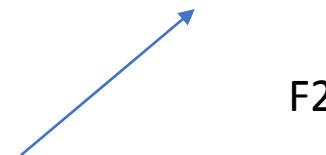
$\theta = \tan^{-1}(F_y/F_x)$ or $\theta = \tan^{-1}(F_y/F_x) + \pi$ based on quadrant

$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$

1) x-component of force is $F_{1x} = -4$ N and y component is $F_{1y} = -3$ N. Write the velocity as vector representation, in magnitude/angle form, and show it graphically



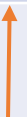
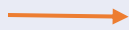
2) A $F_2 = 4.0$ N is making 45 degrees with the x-axis. Find F_{2x} , F_{2y} . Show the vector



3) Both forces are applied to an object. Find $F_{net}(x)$, $F_{net}(y)$ and $|\mathbf{F}|$ by adding the components of each vector

CLASSWORK ON FORCE EQUILIBRIUM

Three forces are acting on an object in equilibrium. Find the missing force F3. Draw the free body diagram and give the answer in the same representation as F1 and F2.

F1(N)		F2(N)		F3(N)		Fnet (N)	
F1x=3.2	F1y=4.5	F2x=-3.6	F2y=2.5			Fnetx=0	Fnety=0
F1x=5	F1y=12	F2x=-5	F2y=-5			Fnetx=0	Fnety=0
F1=4N	$\theta=30$ degrees	F2=4N	$\theta=-30$ degrees			Fnetx=0	Fnety=0
F1=5N	$\theta=90$ degrees	F2=12N	$\theta=0$ degrees			Fnetx=0	Fnety=0
F1 = 3.7 i – 2.6 j		F2 = 4.3 i + 2.8 j				Fnetx=0	Fnety=0
F1 is 2.0 units upward or it is 2.0 units North 		F2 is 2.0 units to the left or F2 is 2.0 units East 				Fnetx=0	Fnety=0

TYPES OF FORCES, THEIR MAGNITUDES AND DIRECTIONS

W = Weight of force of gravity. Direction is always downward. Magnitude is always mg .

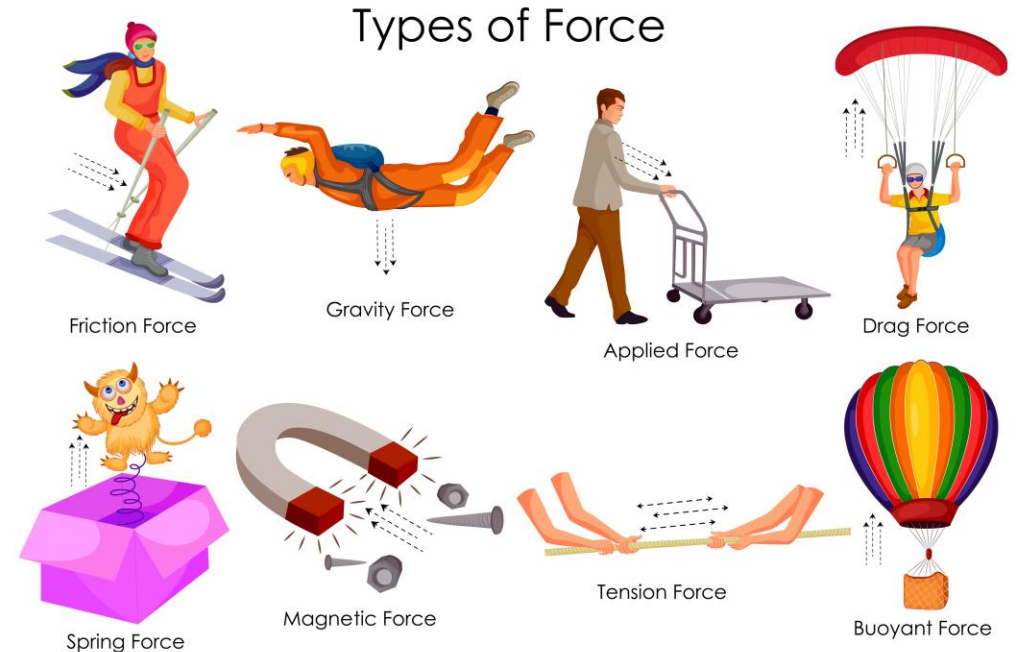
F_n = Normal force. Direction is always perpendicular to the surface. (Magnitude is not always mg)

T = Tension. Direction is always along the string/cable direction. Tension can be given or solved for.

f_s = static friction opposes potential motion direction. Magnitude varies $0 \leq f_s \leq f_s(\max)$ $f_s(\max) = \mu_s F_n$

f_k = usually opposite to the direction of motion. For a system of many objects, it could be in the direction of motion

$F(\text{external})$ = Could be in any direction at any magnitude



KEY STRATEGY: FREE BODY DIAGRAMS

Draw the free body diagram.

Show forces with arrows

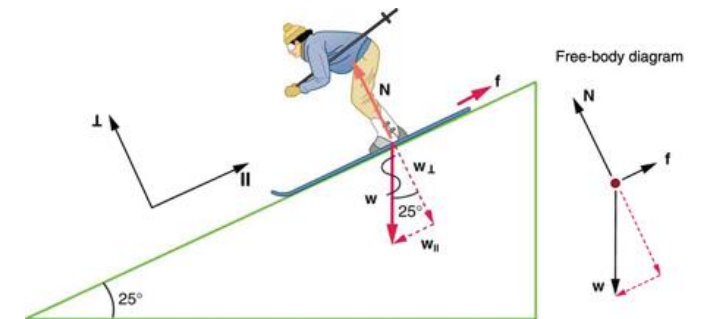
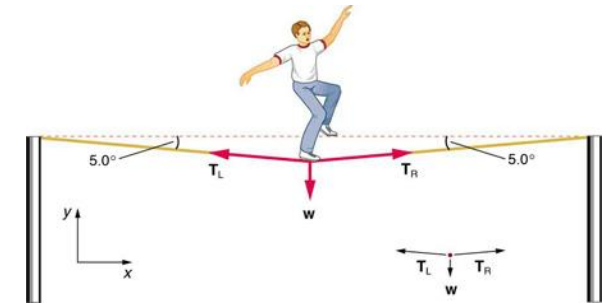
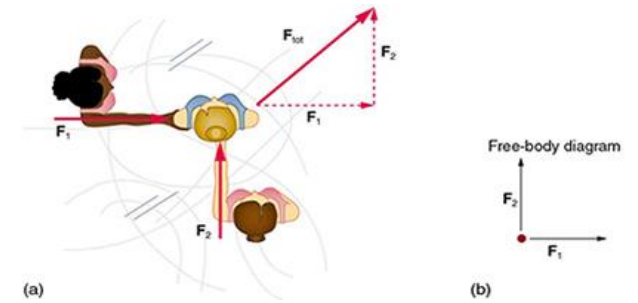
Extract values from the word problem

Find the components of each vector

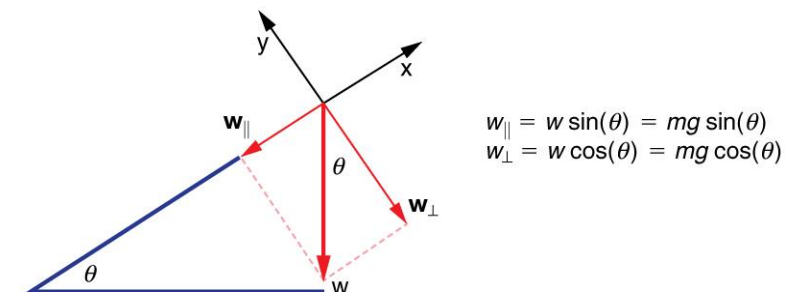
For equilibrium set $F_{netx} = 0$ and $F_{nety} = 0$

For acceleration problems, use $F_{net} = m a$

Solve for the unknown force

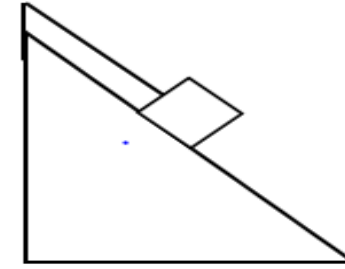


For the inclined plane problems only, you may need to change the coordinate system. Inclined plane problems are solved much easier by taking a diagonal coordinate frame

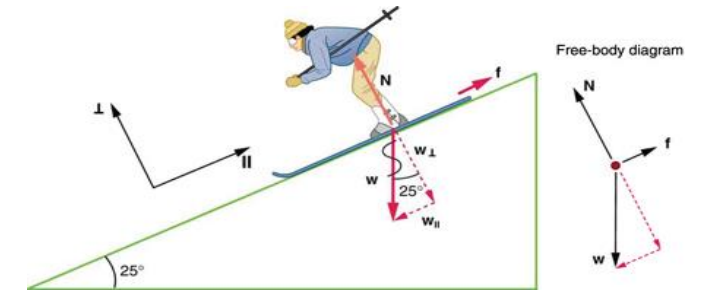


CLASSWORK EQUILIBRIUM

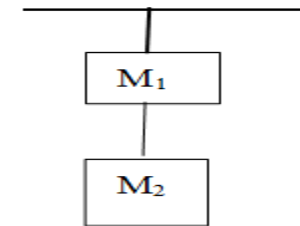
1) A block with 42N weight is balanced using a string on an inclined plane. The angle at the bottom is 40 degrees. Write the equilibrium conditions and solve for the Normal force F_n and the tension T .



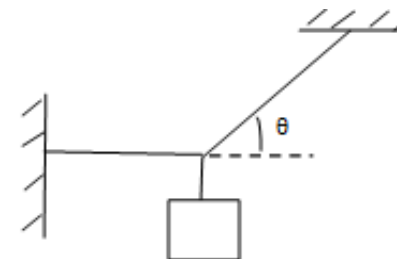
2) A 60 kg skier is moving down on an inclined plane with constant velocity. Inclined plane makes an angle of 25 degrees with the horizontal. a) Find the normal force F_n and the force of friction f_k . What is the μ_s



3) A block $M_1=2.0$ kg, hanging from the ceiling is attached to a second block with mass $M_2= 7.0$ kg. Calculate the tension at each string.



4) A block of mass 0.40 kg is attached to the wire connected to the wall making an angle of 90 degrees with it. It is also attached to the ceiling with the wire making an angle of 53 degrees with the horizontal. Calculate tensions at each wire. Take $\sin(53)=0.8$ and $\cos(53)=0.6$



CLASSWORK ON ACCELERATION

Q1) A 100 kg model rocket is accelerated upward with a 2000 N force applied by its fuel. A few minutes after the launch, there is also 300 N air drag. Find the acceleration of the rocket at that moment. Using drag coefficient as 0.75 and surface area of 2m^2 , calculate its speed.

Q2) A 5.0 kg wood block is on ice being pulled by two equal forces of 20 Newtons applied at the same point. The angle between the forces is 120 degrees. Find the acceleration of the block, neglect friction.

Q3) A 10kg load is pulled diagonally with an unknown external force making 60 degrees with the x-axis against 30N friction force. The pull causes the load to accelerate with 2m/s^2 . What is the external force? What is the coefficient of friction?

Q4) A 60 kg person is in an elevator. A) Calculate the apparent weight of the person (Normal force) when an elevator is going down with acceleration of 3.0 m/s^2 . B) Calculate the apparent weight of the person (Normal force) when an elevator is going up with an acceleration of 3.0 m/s^2 .

ACTIVITY FRICTION FORCE

Open <https://phet.colorado.edu/en/simulations/forces-and-motion-basics>

Choose friction. Click on masses and speed. Set the friction at mid-level.

Adjust the external force until you get a constant speed.

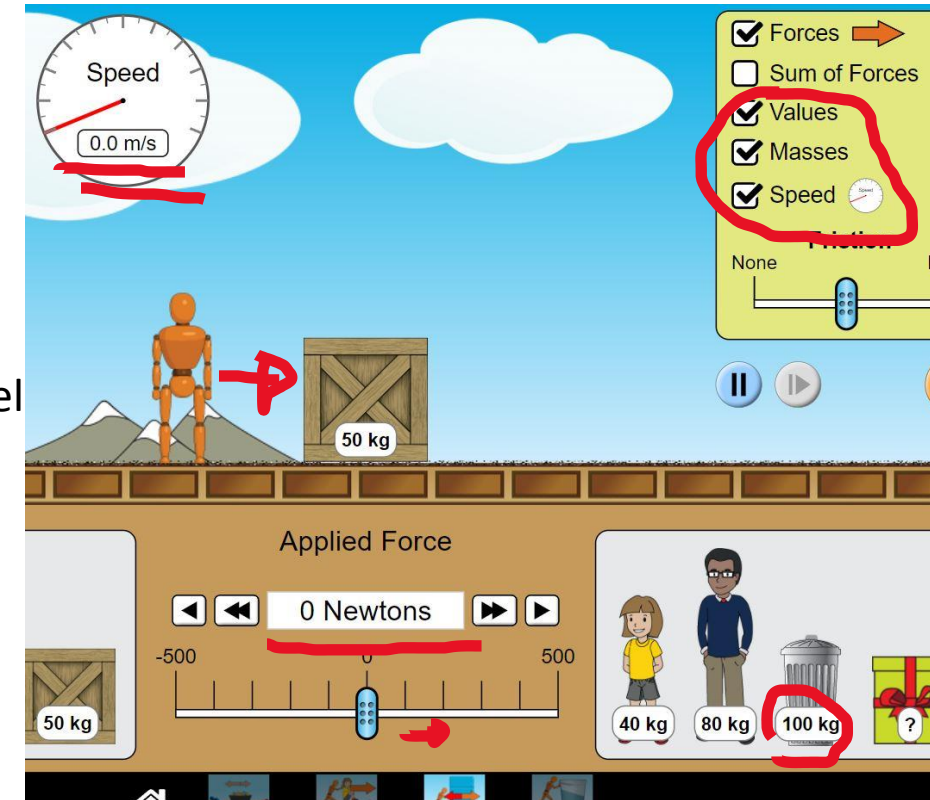
Using the formula $f_k = \mu_k mg$ calculate the friction coefficient.

Double the mass and repeat for the same friction constant.

You should get about the same value. Change the friction level and repeat

Come up with your own numbers and repeat for various masses/friction level

Friction level	Mass(kg)	Fapplied(N)= f_k	μ_k
mid	50		
mid	100		
low	50		
low	100		
high	40		
high	80		



KEY STRATEGIES FOR NEWTON'S THIRD LAW

Newton's third law problems are multiple object problems. Multiple objects forms a system.

For each problem, first draw the free body diagram

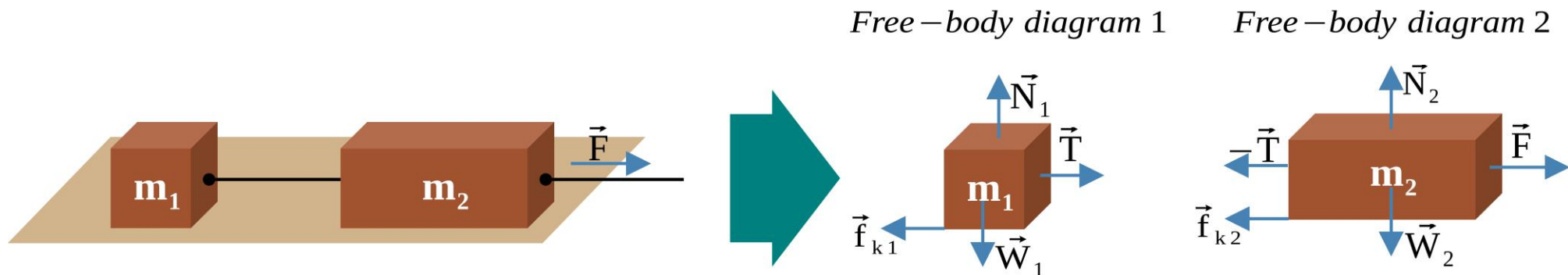
Find the driving force and resistant force (if there is any)

Ignore the internal forces first, treat the problem as a system problem

$F_{net} = F(\text{driving}) - F(\text{resisting})$

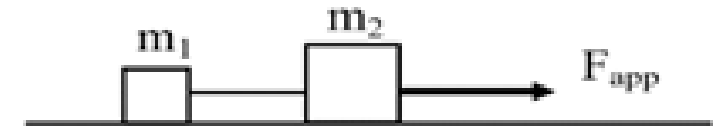
First use $F_{net} = (m_{total} a)$ and solve for the acceleration

Later use free body diagram of one of the objects, substitute for "a" and find the internal force.

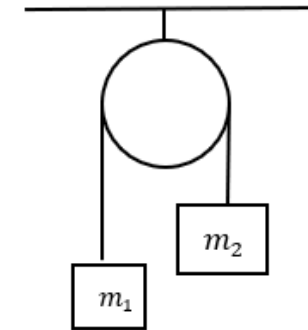


CLASSWORK NEWTON'S THIRD LAW

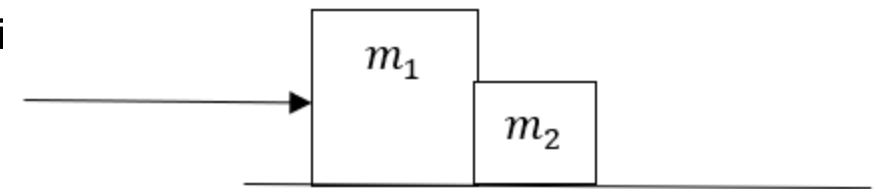
Q1) Two boxes are connected with a string and placed on a frictionless surface. The masses of the boxes are $m_1 = 3 \text{ kg}$ and $m_2 = 7 \text{ kg}$ respectively. The 7 kg box is being pulled by a force of 24 N causing the system to accelerate. Find the acceleration of each box. Find the tension



Q2) Two masses $m_1 = 0.30 \text{ kg}$ and $m_2 = 0.40 \text{ kg}$ are connected by a light string that passes over the frictionless and massless pulley forming an Atwood's machine. The weight of m_2 is the driving force and the weight of m_1 is the resistant force. Find the acceleration of the system and the tension.



Q3) Two blocks are in contact on a frictionless horizontal surface. The blocks are accelerated by a single horizontal force applied to m_1 . Where, $F = 15.6 \text{ N}$, $m_1 = 6.0 \text{ kg}$, and $m_2 = 2.0 \text{ kg}$. Find the acceleration of the system and find the contact force C. B) Find the acceleration if $\mu_k = 0.1$



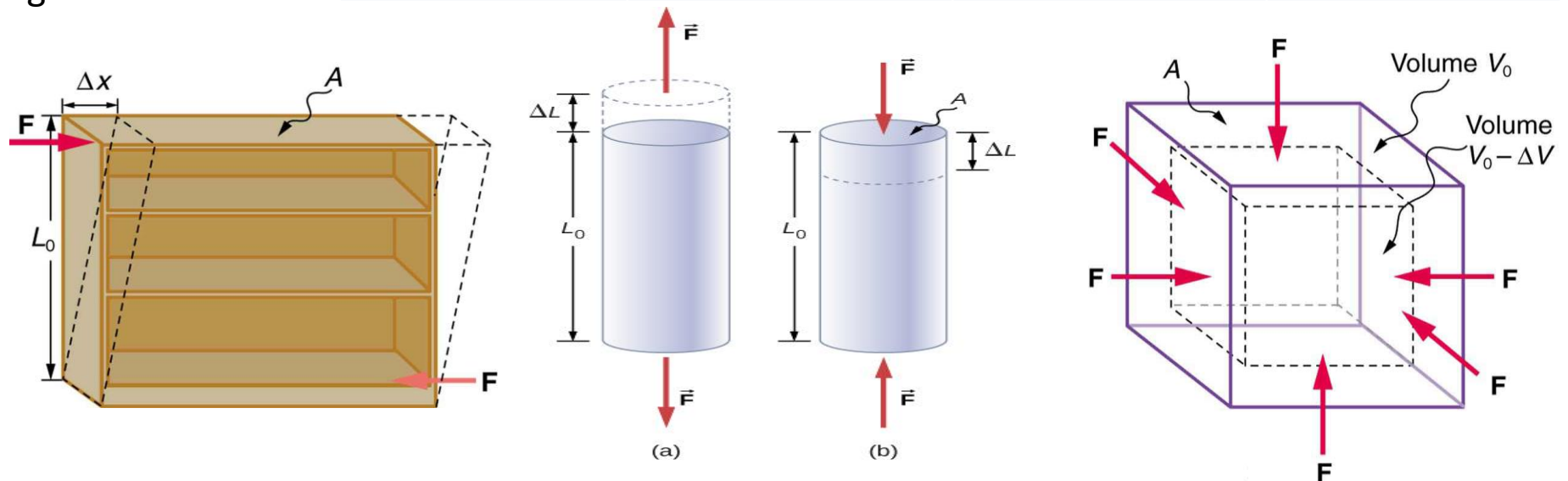
ELASTICITY

Using the table, calculate the deformation of the following under $100,000\text{N/m}^2$ stress.

- 1) Find the change in volume for an Aluminum cube with sides 2 meters long
- 2) Find the change in length for a 0.2 meters long brass
- 3) Find the deformation of a steel cube with length 0.4 meters under shearing force
- 4) Choose a new material using the table and recalculate for 1-2-3.

Material	Young's modulus Y (10^9N/m^2)	Shear modulus S (10^9N/m^2)	Bulk modulus B (10^9N/m^2)
Aluminum	70	25	75
Bone – tension	16	80	8
Brass	90	35	75
Glass	70	20	30
Granite	45	20	45
Lead	16	5	50
Steel	210	80	130

Stress = Modulus * |Strain|
 Strain = Deformation / Original



HOOKE'S LAW ACTIVITY

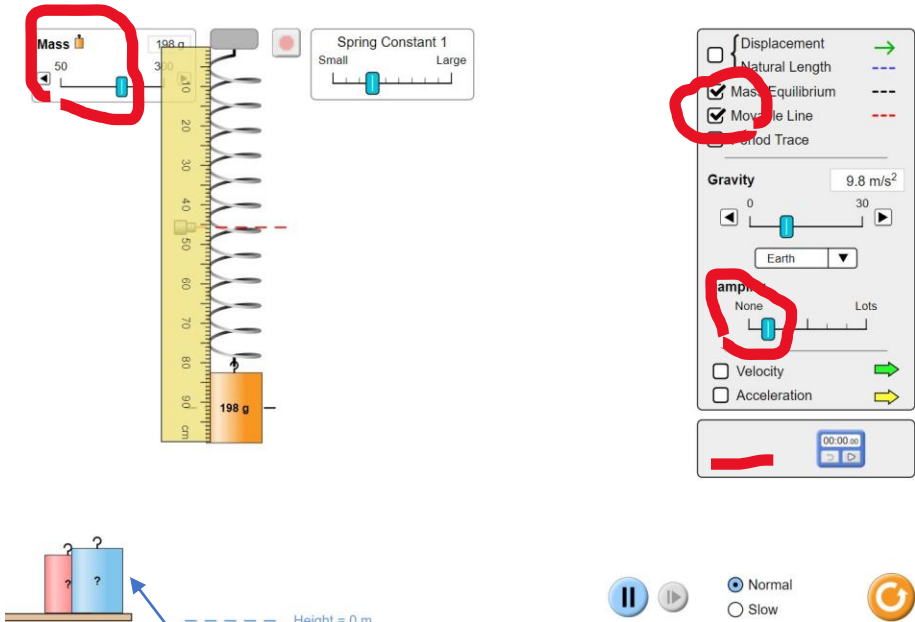
Activity 1) Open https://phet.colorado.edu/sims/html/masses-and-springs/latest/masses-and-springs_en.html

Click on lab. Choose mass equilibrium.

Don't change gravity but put some damping (it is like friction)

Drag the ruler tool, measure the length of the spring without mass on it.

Attach masses based on the table below and measure the new length calculate change in length and using $|F|=k(L-L_0)$ find the coefficient k
 $F = mg$ however mass is in grams so convert it to kg. L at zero mass is your L_0 .
 You will use it to find change in length. "k" can't be calculated at that value.



M(kg)	F(N)	L (m)	L-Lo (m)	k
0	0			
0.050				
0.100				
0.150				

Activity 2) Attach one of the mysterious objects. Using the "k" value calculate their masses.

REFERENCES

- Open Stax
- Adobe stock photo
- PhET